A.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. A real number is ________ if it can be written as the ratio $\frac{p}{q}$ of two integers, where $q \neq 0$.
2. ________ numbers have infinite nonrepeating decimal representations.
3. The point 0 on the real number line is called the ________.
4. The distance between the origin and a point representing a real number on the real number line is the ________ ________ of the real number.
5. A number that can be written as the product of two or more prime numbers is called a ________ number.
6. An integer that has exactly two positive factors, the integer itself and 1, is called a ________ number.
7. An algebraic expression is a collection of letters called ________ and real numbers called ________.
8. The ________ of an algebraic expression are those parts separated by addition.
9. The numerical factor of a variable term is the ________ of the variable term.
10. The ________ ________ states that if $ab = 0$, then $a = 0$ or $b = 0$.

SKILLS AND APPLICATIONS

In Exercises 11–16, determine which numbers in the set are (a) natural numbers, (b) whole numbers, (c) integers, (d) rational numbers, and (e) irrational numbers.

11. $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, 2, -11\right\}$
12. $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -3, 12, 5\right\}$
13. $\{2.01, 0.666\ldots, -13, 0.010110111\ldots, 1, -6\}$
14. $\{2.3030030003\ldots, 0.7575, -4.63, \sqrt{10}, -75, 4\}$
15. $\left\{-\pi, -\frac{1}{3}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22\right\}$
16. $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 7, -11.1, 13\right\}$

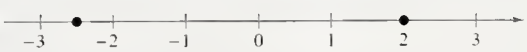
In Exercises 17 and 18, plot the real numbers on the real number line.

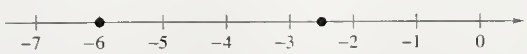
17. (a) 3 (b) $\frac{7}{2}$ (c) $-\frac{5}{2}$ (d) -5.2
18. (a) 8.5 (b) $\frac{4}{3}$ (c) -4.75 (d) $-\frac{8}{3}$

In Exercises 19–22, use a calculator to find the decimal form of the rational number. If it is a nonterminating decimal, write the repeating pattern.

19. $\frac{5}{8}$
20. $\frac{1}{3}$
21. $\frac{41}{333}$
22. $\frac{6}{11}$

In Exercises 23 and 24, approximate the numbers and place the correct symbol (< or >) between them.

23. 

24. 

In Exercises 25–30, plot the two real numbers on the real number line. Then place the appropriate inequality symbol (< or >) between them.

25. $-4, -8$
26. $-3.5, 1$
27. $\frac{3}{2}, 7$
28. $1, \frac{16}{3}$
29. $\frac{5}{6}, \frac{2}{3}$
30. $-\frac{8}{7}, -\frac{3}{7}$

In Exercises 31–42, (a) give a verbal description of the subset of real numbers represented by the inequality or the interval, (b) sketch the subset on the real number line, and (c) state whether the interval is bounded or unbounded.

31. $x \leq 5$
32. $x \geq -2$
33. $x < 0$
34. $x > 3$
35. $[4, \infty)$
36. $(-\infty, 2)$
37. $-2 < x < 2$
38. $0 \leq x \leq 5$
39. $-1 \leq x < 0$
40. $0 < x \leq 6$
41. $[-2, 5)$
42. $(-1, 2]$

In Exercises 43–50, use inequality notation and interval notation to describe the set.

43. y is nonnegative.
44. y is no more than 25.
45. x is greater than -2 and at most 4.
46. y is at least -6 and less than 0.
47. t is at least 10 and at most 22.
48. k is less than 5 but no less than -3.
49. The dog's weight W is more than 65 pounds.
50. The annual rate of inflation r is expected to be at least 2.5% but no more than 5%.

In Exercises 51–60, evaluate the expression.

51. $|-10|$

52. $|0|$

53. $|3 - 8|$

54. $|4 - 1|$

55. $|-1| - |-2|$

56. $-3 - |-3|$

57. $\dfrac{-5}{|-5|}$

58. $-3|-3|$

59. $\dfrac{|x + 2|}{x + 2}, \quad x < -2$

60. $\dfrac{|x - 1|}{x - 1}, \quad x > 1$

In Exercises 61–66, place the correct symbol (<, >, or =) between the two real numbers.

61. $|-3| \quad \square \quad -|-3|$

62. $|-4| \quad \square \quad |4|$

63. $-5 \quad \square \quad -|5|$

64. $-|-6| \quad \square \quad |-6|$

65. $-|-2| \quad \square \quad -|2|$

66. $-(-2) \quad \square \quad -2$

In Exercises 67–72, find the distance between *a* and *b*.

67. $a = 126,\ b = 75$

68. $a = -126,\ b = -75$

69. $a = -\frac{5}{2},\ b = 0$

70. $a = \frac{1}{4},\ b = \frac{11}{4}$

71. $a = \frac{16}{5},\ b = \frac{112}{75}$

72. $a = 9.34,\ b = -5.65$

In Exercises 73–78, use absolute value notation to describe the situation.

73. The distance between x and 5 is no more than 3.

74. The distance between x and -10 is at least 6.

75. y is at least six units from 0.

76. y is at most two units from a.

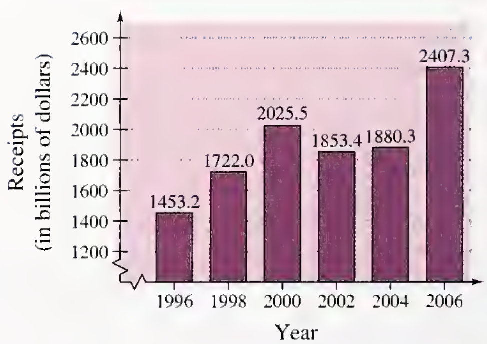
77. While traveling on the Pennsylvania Turnpike, you pass milepost 57 near Pittsburgh, then milepost 236 near Gettysburg. How many miles do you travel during that time period?

78. The temperature in Bismarck, North Dakota was 60°F at noon, then 23°F at midnight. What was the change in temperature over the 12-hour period?

BUDGET VARIANCE In Exercises 79–82, the accounting department of a sports drink bottling company is checking to see whether the actual expenses of a department differ from the budgeted expenses by more than \$500 or by more than 5%. Fill in the missing parts of the table, and determine whether each actual expense passes the "budget variance test."

	Budgeted Expense, b	*Actual Expense, a*	$\lvert a - b \rvert$	$0.05b$
79. Wages	\$112,700	\$113,356		
80. Utilities	\$9,400	\$9,772		
81. Taxes	\$37,640	\$37,335		
82. Insurance	\$2,575	\$2,613		

FEDERAL DEFICIT In Exercises 83–88, use the bar graph, which shows the receipts of the federal government (in billions of dollars) for selected years from 1996 through 2006. In each exercise you are given the expenditures of the federal government. Find the magnitude of the surplus or deficit for the year. (Source: U.S. Office of Management and Budget)



Year	*Receipts*	*Expenditures*	*|Receipts − Expenditures|*
83. 1996		\$1560.6 billion	
84. 1998		\$1652.7 billion	
85. 2000		\$1789.2 billion	
86. 2002		\$2011.2 billion	
87. 2004		\$2293.0 billion	
88. 2006		\$2655.4 billion	

In Exercises 89–94, identify the terms. Then identify the coefficients of the variable terms of the expression.

89. $7x + 4$

90. $6x^3 - 5x$

91. $\sqrt{3}x^2 - 8x - 11$

92. $3\sqrt{3}x^2 + 1$

93. $4x^3 + \dfrac{x}{2} - 5$

94. $3x^4 - \dfrac{x^2}{4}$

In Exercises 95–100, evaluate the expression for each value of x. (If not possible, state the reason.)

Expression	*Values*	
95. $4x - 6$	(a) $x = -1$	(b) $x = 0$
96. $9 - 7x$	(a) $x = -3$	(b) $x = 3$
97. $x^2 - 3x + 4$	(a) $x = -2$	(b) $x = 2$
98. $-x^2 + 5x - 4$	(a) $x = -1$	(b) $x = 1$
99. $\dfrac{x+1}{x-1}$	(a) $x = 1$	(b) $x = -1$
100. $\dfrac{x}{x+2}$	(a) $x = 2$	(b) $x = -2$

In Exercises 101–112, identify the rule(s) of algebra illustrated by the statement.

101. $x + 9 = 9 + x$

102. $2\left(\frac{1}{2}\right) = 1$

103. $\dfrac{1}{h+6}(h + 6) = 1, \quad h \neq -6$

104. $(x + 3) - (x + 3) = 0$

105. $2(x + 3) = 2 \cdot x + 2 \cdot 3$

106. $(z - 2) + 0 = z - 2$

107. $1 \cdot (1 + x) = 1 + x$

108. $(z + 5)x = z \cdot x + 5 \cdot x$

109. $x + (y + 10) = (x + y) + 10$

110. $x(3y) = (x \cdot 3)y = (3x)y$

111. $3(t - 4) = 3 \cdot t - 3 \cdot 4$

112. $\frac{1}{7}(7 \cdot 12) = \left(\frac{1}{7} \cdot 7\right)12 = 1 \cdot 12 = 12$

In Exercises 113–120, perform the operation(s). (Write fractional answers in simplest form.)

113. $\frac{3}{16} + \frac{5}{16}$

114. $\frac{6}{7} - \frac{4}{7}$

115. $\frac{5}{8} - \frac{5}{12} + \frac{1}{6}$

116. $\frac{10}{11} + \frac{6}{33} - \frac{13}{66}$

117. $12 \div \frac{1}{4}$

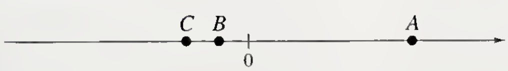
118. $-\left(6 \cdot \frac{4}{8}\right)$

119. $\dfrac{2x}{3} - \dfrac{x}{4}$

120. $\dfrac{5x}{6} \cdot \dfrac{2}{9}$

EXPLORATION

In Exercises 121 and 122, use the real numbers A, B, and C shown on the number line. Determine the sign of each expression.



121. (a) $-A$
(b) $B - A$

122. (a) $-C$
(b) $A - C$

123. CONJECTURE

(a) Use a calculator to complete the table.

n	1	0.5	0.01	0.0001	0.000001
$5/n$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n approaches 0.

124. CONJECTURE

(a) Use a calculator to complete the table.

n	1	10	100	10,000	100,000
$5/n$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n increases without bound.

TRUE OR FALSE? In Exercises 125–128, determine whether the statement is true or false. Justify your answer.

125. If $a > 0$ and $b < 0$, then $a - b > 0$.

126. If $a > 0$ and $b < 0$, then $ab > 0$.

127. If $a < b$, then $\dfrac{1}{a} < \dfrac{1}{b}$, where $a \neq 0$ and $b \neq 0$.

128. Because $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$, then $\dfrac{c}{a+b} = \dfrac{c}{a} + \dfrac{c}{b}$.

129. THINK ABOUT IT Consider $|u + v|$ and $|u| + |v|$, where $u \neq 0$ and $v \neq 0$.

(a) Are the values of the expressions always equal? If not, under what conditions are they unequal?

(b) If the two expressions are not equal for certain values of u and v, is one of the expressions always greater than the other? Explain.

130. THINK ABOUT IT Is there a difference between saying that a real number is positive and saying that a real number is nonnegative? Explain.

131. THINK ABOUT IT Because every even number is divisible by 2, is it possible that there exist any even prime numbers? Explain.

132. THINK ABOUT IT Is it possible for a real number to be both rational and irrational? Explain.

133. WRITING Can it ever be true that $|a| = -a$ for a real number a? Explain.

134. CAPSTONE Describe the differences among the sets of natural numbers, whole numbers, integers, rational numbers, and irrational numbers.

A.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. In the exponential form a^n, n is the ________ and a is the ________.
2. A convenient way of writing very large or very small numbers is called ________ ________.
3. One of the two equal factors of a number is called a ________ ________ of the number.
4. The ________ ________ ________ of a number a is the nth root that has the same sign as a, and is denoted by $\sqrt[n]{a}$.
5. In the radical form $\sqrt[n]{a}$, the positive integer n is called the ________ of the radical and the number a is called the ________.
6. When an expression involving radicals has all possible factors removed, radical-free denominators, and a reduced index, it is in ________ ________.
7. Radical expressions can be combined (added or subtracted) if they are ________ ________.
8. The expressions $a + b\sqrt{m}$ and $a - b\sqrt{m}$ are ________ of each other.
9. The process used to create a radical-free denominator is known as ________ the denominator.
10. In the expression $b^{m/n}$, m denotes the ________ to which the base is raised and n denotes the ________ or root to be taken.

SKILLS AND APPLICATIONS

In Exercises 11–18, evaluate each expression.

11. (a) $3^2 \cdot 3$ (b) $3 \cdot 3^3$
12. (a) $\dfrac{5^5}{5^2}$ (b) $\dfrac{3^2}{3^4}$
13. (a) $(3^3)^0$ (b) -3^2
14. (a) $(2^3 \cdot 3^2)^2$ (b) $\left(-\frac{3}{5}\right)^3\left(\frac{5}{3}\right)^2$
15. (a) $\dfrac{3}{3^{-4}}$ (b) $48(-4)^{-3}$
16. (a) $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$ (b) $(-2)^0$
17. (a) $2^{-1} + 3^{-1}$ (b) $(2^{-1})^{-2}$
18. (a) $3^{-1} + 2^{-2}$ (b) $(3^{-2})^2$

In Exercises 19–22, use a calculator to evaluate the expression. (If necessary, round your answer to three decimal places.)

19. $(-4)^3(5^2)$
20. $(8^{-4})(10^3)$
21. $\dfrac{3^6}{7^3}$
22. $\dfrac{4^3}{3^{-4}}$

In Exercises 23–30, evaluate the expression for the given value of x.

23. $-3x^3$, $x = 2$
24. $7x^{-2}$, $x = 4$
25. $6x^0$, $x = 10$
26. $5(-x)^3$, $x = 3$
27. $2x^3$, $x = -3$
28. $-3x^4$, $x = -2$
29. $-20x^2$, $x = -\frac{1}{2}$
30. $12(-x)^3$, $x = -\frac{1}{3}$

In Exercises 31–38, simplify each expression.

31. (a) $(-5z)^3$ (b) $5x^4(x^2)$
32. (a) $(3x)^2$ (b) $(4x^3)^0$, $x \neq 0$
33. (a) $6y^2(2y^0)^2$ (b) $\dfrac{3x^5}{x^3}$
34. (a) $(-z)^3(3z^4)$ (b) $\dfrac{25y^8}{10y^4}$
35. (a) $\dfrac{7x^2}{x^3}$ (b) $\dfrac{12(x+y)^3}{9(x+y)}$
36. (a) $\dfrac{r^4}{r^6}$ (b) $\left(\dfrac{4}{y}\right)^3\left(\dfrac{3}{y}\right)^4$
37. (a) $[(x^2y^{-2})^{-1}]^{-1}$ (b) $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3$
38. (a) $(6x^7)^0$, $x \neq 0$ (b) $(5x^2z^6)^3(5x^2z^6)^{-3}$

In Exercises 39–44, rewrite each expression with positive exponents and simplify.

39. (a) $(x + 5)^0$, $x \neq -5$ (b) $(2x^2)^{-2}$
40. (a) $(2x^5)^0$, $x \neq 0$ (b) $(z + 2)^{-3}(z + 2)^{-1}$
41. (a) $(-2x^2)^3(4x^3)^{-1}$ (b) $\left(\dfrac{x}{10}\right)^{-1}$
42. (a) $(4y^{-2})(8y^4)$ (b) $\left(\dfrac{x^{-3}y^4}{5}\right)^{-3}$
43. (a) $3^n \cdot 3^{2n}$ (b) $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3$
44. (a) $\dfrac{x^2 \cdot x^n}{x^3 \cdot x^n}$ (b) $\left(\dfrac{a^{-3}}{b^{-3}}\right)\left(\dfrac{a}{b}\right)^3$

In Exercises 45–52, write the number in scientific notation.

45. 10,250.4 **46.** $-7{,}280{,}000$

47. -0.000125 **48.** 0.00052

49. Land area of Earth: 57,300,000 square miles

50. Light year: 9,460,000,000,000 kilometers

51. Relative density of hydrogen: 0.0000899 gram per cubic centimeter

52. One micron (millionth of a meter): 0.00003937 inch

In Exercises 53–60, write the number in decimal notation.

53. 1.25×10^5 **54.** -1.801×10^5

55. -2.718×10^{-3} **56.** 3.14×10^{-4}

57. Interior temperature of the sun: 1.5×10^7 degrees Celsius

58. Charge of an electron: 1.6022×10^{-19} coulomb

59. Width of a human hair: 9.0×10^{-5} meter

60. Gross domestic product of the United States in 2007: 1.3743021×10^{13} dollars (Source: U.S. Department of Commerce)

In Exercises 61 and 62, evaluate each expression without using a calculator.

61. (a) $(2.0 \times 10^9)(3.4 \times 10^{-4})$
(b) $(1.2 \times 10^7)(5.0 \times 10^{-3})$

62. (a) $\dfrac{6.0 \times 10^8}{3.0 \times 10^{-3}}$ (b) $\dfrac{2.5 \times 10^{-3}}{5.0 \times 10^2}$

In Exercises 63 and 64, use a calculator to evaluate each expression. (Round your answer to three decimal places.)

63. (a) $750\left(1 + \dfrac{0.11}{365}\right)^{800}$
(b) $\dfrac{67{,}000{,}000 + 93{,}000{,}000}{0.0052}$

64. (a) $(9.3 \times 10^6)^3(6.1 \times 10^{-4})$ (b) $\dfrac{(2.414 \times 10^4)^6}{(1.68 \times 10^5)^5}$

In Exercises 65–70, evaluate each expression without using a calculator.

65. (a) $\sqrt{9}$ (b) $\sqrt[3]{\frac{27}{8}}$

66. (a) $27^{1/3}$ (b) $36^{3/2}$

67. (a) $32^{-3/5}$ (b) $\left(\frac{16}{81}\right)^{-3/4}$

68. (a) $100^{-3/2}$ (b) $\left(\frac{9}{4}\right)^{-1/2}$

69. (a) $\left(-\dfrac{1}{64}\right)^{-1/3}$ (b) $\left(\dfrac{1}{\sqrt{32}}\right)^{-2/5}$

70. (a) $\left(-\dfrac{125}{27}\right)^{-1/3}$ (b) $-\left(\dfrac{1}{125}\right)^{-4/3}$

In Exercises 71–76, use a calculator to approximate the number. (Round your answer to three decimal places.)

71. (a) $\sqrt{57}$ (b) $\sqrt[5]{-27^3}$

72. (a) $\sqrt[3]{45^2}$ (b) $\sqrt[6]{125}$

73. (a) $(-12.4)^{-1.8}$ (b) $(5\sqrt{3})^{-2.5}$

74. (a) $\dfrac{7 - (4.1)^{-3.2}}{2}$ (b) $\left(\dfrac{13}{3}\right)^{-3/2} - \left(-\dfrac{3}{2}\right)^{13/3}$

75. (a) $\sqrt{4.5 \times 10^9}$ (b) $\sqrt[3]{6.3 \times 10^4}$

76. (a) $(2.65 \times 10^{-4})^{1/3}$ (b) $\sqrt{9 \times 10^{-4}}$

In Exercises 77 and 78, use the properties of radicals to simplify each expression.

77. (a) $(\sqrt[5]{2})^5$ (b) $\sqrt[5]{96x^5}$

78. (a) $\sqrt{12} \cdot \sqrt{3}$ (b) $\sqrt[4]{(3x^2)^4}$

In Exercises 79–90, simplify each radical expression.

79. (a) $\sqrt{20}$ (b) $\sqrt[3]{128}$

80. (a) $\sqrt[3]{\frac{16}{27}}$ (b) $\sqrt{\frac{75}{4}}$

81. (a) $\sqrt{72x^3}$ (b) $\sqrt{\dfrac{18^2}{z^3}}$

82. (a) $\sqrt{54xy^4}$ (b) $\sqrt{\dfrac{32a^4}{b^2}}$

83. (a) $\sqrt[3]{16x^5}$ (b) $\sqrt{75x^2y^{-4}}$

84. (a) $\sqrt[4]{3x^4y^2}$ (b) $\sqrt[5]{160x^8z^4}$

85. (a) $2\sqrt{50} + 12\sqrt{8}$ (b) $10\sqrt{32} - 6\sqrt{18}$

86. (a) $4\sqrt{27} - \sqrt{75}$ (b) $\sqrt[3]{16} + 3\sqrt[3]{54}$

87. (a) $5\sqrt{x} - 3\sqrt{x}$ (b) $-2\sqrt{9y} + 10\sqrt{y}$

88. (a) $8\sqrt{49x} - 14\sqrt{100x}$
(b) $-3\sqrt{48x^2} + 7\sqrt{75x^2}$

89. (a) $3\sqrt{x+1} + 10\sqrt{x+1}$
(b) $7\sqrt{80x} - 2\sqrt{125x}$

90. (a) $-\sqrt{x^3 - 7} + 5\sqrt{x^3 - 7}$
(b) $11\sqrt{245x^3} - 9\sqrt{45x^3}$

In Exercises 91–94, complete the statement with <, =, or >.

91. $\sqrt{5} + \sqrt{3}$ ▭ $\sqrt{5 + 3}$ **92.** $\sqrt{\dfrac{3}{11}}$ ▭ $\dfrac{\sqrt{3}}{\sqrt{11}}$

93. 5 ▭ $\sqrt{3^2 + 2^2}$ **94.** 5 ▭ $\sqrt{3^2 + 4^2}$

In Exercises 95–98, rationalize the denominator of the expression. Then simplify your answer.

95. $\dfrac{1}{\sqrt{3}}$ **96.** $\dfrac{8}{\sqrt[3]{2}}$

97. $\dfrac{5}{\sqrt{14} - 2}$ **98.** $\dfrac{3}{\sqrt{5} + \sqrt{6}}$

In Exercises 99–102, rationalize the numerator of the expression. Then simplify your answer.

99. $\dfrac{\sqrt{8}}{2}$

100. $\dfrac{\sqrt{2}}{3}$

101. $\dfrac{\sqrt{5}+\sqrt{3}}{3}$

102. $\dfrac{\sqrt{7}-3}{4}$

In Exercises 103–110, fill in the missing form of the expression.

	Radical Form	*Rational Exponent Form*
103.	$\sqrt{2.5}$	
104.	$\sqrt[3]{64}$	
105.		$81^{1/4}$
106.		$-(144^{1/2})$
107.	$\sqrt[3]{-216}$	
108.		$(-243)^{1/5}$
109.	$(\sqrt[4]{81})^3$	
110.		$16^{5/4}$

In Exercises 111–114, perform the operations and simplify.

111. $\dfrac{(2x^2)^{3/2}}{2^{1/2}x^4}$

112. $\dfrac{x^{4/3}y^{2/3}}{(xy)^{1/3}}$

113. $\dfrac{x^{-3}\cdot x^{1/2}}{x^{3/2}\cdot x^{-1}}$

114. $\dfrac{5^{-1/2}\cdot 5x^{5/2}}{(5x)^{3/2}}$

In Exercises 115 and 116, reduce the index of each radical.

115. (a) $\sqrt[4]{3^2}$ (b) $\sqrt[6]{(x+1)^4}$

116. (a) $\sqrt[6]{x^3}$ (b) $\sqrt[4]{(3x^2)^4}$

In Exercises 117 and 118, write each expression as a single radical. Then simplify your answer.

117. (a) $\sqrt{\sqrt{32}}$ (b) $\sqrt{\sqrt[4]{2x}}$

118. (a) $\sqrt{\sqrt{243(x+1)}}$ (b) $\sqrt{\sqrt[3]{10a^7b}}$

119. **PERIOD OF A PENDULUM** The period T (in seconds) of a pendulum is $T = 2\pi\sqrt{L/32}$, where L is the length of the pendulum (in feet). Find the period of a pendulum whose length is 2 feet.

120. **EROSION** A stream of water moving at the rate of v feet per second can carry particles of size $0.03\sqrt{v}$ inches. Find the size of the largest particle that can be carried by a stream flowing at the rate of $\frac{3}{4}$ foot per second.

121. **MATHEMATICAL MODELING** A funnel is filled with water to a height of h centimeters. The formula

$$t = 0.03[12^{5/2} - (12-h)^{5/2}],\quad 0 \le h \le 12$$

represents the amount of time t (in seconds) that it will take for the funnel to empty.

(a) Use the table feature of a graphing utility to find the times required for the funnel to empty for water heights of $h = 0, h = 1, h = 2, \ldots h = 12$ centimeters.

(b) What value does t appear to be approaching as the height of the water becomes closer and closer to 12 centimeters?

122. **SPEED OF LIGHT** The speed of light is approximately 11,180,000 miles per minute. The distance from the sun to Earth is approximately 93,000,000 miles. Find the time for light to travel from the sun to Earth.

EXPLORATION

TRUE OR FALSE? In Exercises 123 and 124, determine whether the statement is true or false. Justify your answer.

123. $\dfrac{x^{k+1}}{x} = x^k$

124. $(a^n)^k = a^{n^k}$

125. Verify that $a^0 = 1, a \neq 0$. (*Hint:* Use the property of exponents $a^m/a^n = a^{m-n}$.)

126. Explain why each of the following pairs is not equal.

(a) $(3x)^{-1} \neq \dfrac{3}{x}$ (b) $y^3\cdot y^2 \neq y^6$

(c) $(a^2b^3)^4 \neq a^6b^7$ (d) $(a+b)^2 \neq a^2+b^2$

(e) $\sqrt{4x^2} \neq 2x$ (f) $\sqrt{2}+\sqrt{3} \neq \sqrt{5}$

127. **THINK ABOUT IT** Is 52.7×10^5 written in scientific notation? Why or why not?

128. List all possible digits that occur in the units place of the square of a positive integer. Use that list to determine whether $\sqrt{5233}$ is an integer.

129. **THINK ABOUT IT** Square the real number $5/\sqrt{3}$ and note that the radical is eliminated from the denominator. Is this equivalent to rationalizing the denominator? Why or why not?

130. **CAPSTONE**

(a) Explain how to simplify the expression $(3x^3y^{-2})^{-2}$.

(b) Is the expression $\sqrt{\dfrac{4}{x^3}}$ in simplest form? Why or why not?

The symbol indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

The symbol indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

A.3 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. For the polynomial $a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1x + a_0$, $a_n \neq 0$, the degree is ________, the leading coefficient is ________, and the constant term is ________.
2. A polynomial in x in standard form is written with ________ powers of x.
3. A polynomial with one term is called a ________, while a polynomial with two terms is called a ________, and a polynomial with three terms is called a ________.
4. To add or subtract polynomials, add or subtract the ________ ________ by adding their coefficients.
5. The letters in "FOIL" stand for the following.
   F ________ O ________ I ________ L ________
6. The process of writing a polynomial as a product is called ________.
7. A polynomial is ________ ________ when each of its factors is prime.
8. A polynomial $u^2 + 2uv + v^2$ is called a ________ ________ ________.

SKILLS AND APPLICATIONS

In Exercises 9–14, match the polynomial with its description. [The polynomials are labeled (a), (b), (c), (d), (e), and (f).]

(a) $3x^2$ (b) $1 - 2x^3$
(c) $x^3 + 3x^2 + 3x + 1$ (d) 12
(e) $-3x^5 + 2x^3 + x$ (f) $\frac{2}{3}x^4 + x^2 + 10$

9. A polynomial of degree 0
10. A trinomial of degree 5
11. A binomial with leading coefficient -2
12. A monomial of positive degree
13. A trinomial with leading coefficient $\frac{2}{3}$
14. A third-degree polynomial with leading coefficient 1

In Exercises 15–18, write a polynomial that fits the description. (There are many correct answers.)

15. A third-degree polynomial with leading coefficient -2
16. A fifth-degree polynomial with leading coefficient 6
17. A fourth-degree binomial with a negative leading coefficient
18. A third-degree binomial with an even leading coefficient

In Exercises 19–30, (a) write the polynomial in standard form, (b) identify the degree and leading coefficient of the polynomial, and (c) state whether the polynomial is a monomial, a binomial, or a trinomial.

19. $14x - \frac{1}{2}x^5$
20. $2x^2 - x + 1$
21. $x^2 - 4 - 3x^4$
22. $7x$
23. $3 - x^6$
24. $-y + 25y^2 + 1$
25. 3
26. $-8 + t^2$
27. $1 + 6x^4 - 4x^5$
28. $3 + 2x$
29. $4x^3y$
30. $-x^5y + 2x^2y^2 + xy^4$

In Exercises 31–36, determine whether the expression is a polynomial. If so, write the polynomial in standard form.

31. $2x - 3x^3 + 8$
32. $5x^4 - 2x^2 + x^{-2}$
33. $\dfrac{3x + 4}{x}$
34. $\dfrac{x^2 + 2x - 3}{2}$
35. $y^2 - y^4 + y^3$
36. $y^4 - \sqrt{y}$

In Exercises 37–54, perform the operation and write the result in standard form.

37. $(6x + 5) - (8x + 15)$
38. $(2x^2 + 1) - (x^2 - 2x + 1)$
39. $-(t^3 - 1) + (6t^3 - 5t)$
40. $-(5x^2 - 1) - (-3x^2 + 5)$
41. $(15x^2 - 6) - (-8.3x^3 - 14.7x^2 - 17)$
42. $(15.6w^4 - 14w - 17.4) - (16.9w^4 - 9.2w + 13)$
43. $5z - [3z - (10z + 8)]$
44. $(y^3 + 1) - [(y^2 + 1) + (3y - 7)]$
45. $3x(x^2 - 2x + 1)$
46. $y^2(4y^2 + 2y - 3)$
47. $-5z(3z - 1)$
48. $(-3x)(5x + 2)$
49. $(1 - x^3)(4x)$
50. $-4x(3 - x^3)$
51. $(1.5t^2 + 5)(-3t)$
52. $(2 - 3.5y)(2y^3)$
53. $-2x(0.1x + 17)$
54. $6y(5 - \frac{3}{8}y)$

In Exercises 55–92, multiply or find the special product.

55. $(x + 3)(x + 4)$
56. $(x - 5)(x + 10)$
57. $(3x - 5)(2x + 1)$
58. $(7x - 2)(4x - 3)$
59. $(x + 10)(x - 10)$
60. $(2x + 3)(2x - 3)$
61. $(x + 2y)(x - 2y)$
62. $(4a + 5b)(4a - 5b)$
63. $(2x + 3)^2$
64. $(5 - 8x)^2$

65. $(x + 1)^3$
66. $(x - 2)^3$
67. $(2x - y)^3$
68. $(3x + 2y)^3$
69. $(4x^3 - 3)^2$
70. $(8x + 3)^2$
71. $(x^2 - x + 1)(x^2 + x + 1)$
72. $(x^2 + 3x - 2)(x^2 - 3x - 2)$
73. $(-x^2 + x - 5)(3x^2 + 4x + 1)$
74. $(2x^2 - x + 4)(x^2 + 3x + 2)$
75. $[(m - 3) + n][(m - 3) - n]$
76. $[(x - 3y) + z][(x - 3y) - z]$
77. $[(x - 3) + y]^2$
78. $[(x + 1) - y]^2$
79. $(2r^2 - 5)(2r^2 + 5)$
80. $(3a^3 - 4b^2)(3a^3 + 4b^2)$
81. $(\frac{1}{4}x - 5)^2$
82. $(\frac{3}{5}t + 4)^2$
83. $(\frac{1}{5}x - 3)(\frac{1}{5}x + 3)$
84. $(3x + \frac{1}{6})(3x - \frac{1}{6})$
85. $(2.4x + 3)^2$
86. $(1.8y - 5)^2$
87. $(1.5x - 4)(1.5x + 4)$
88. $(2.5y + 3)(2.5y - 3)$
89. $5x(x + 1) - 3x(x + 1)$
90. $(2x - 1)(x + 3) + 3(x + 3)$
91. $(u + 2)(u - 2)(u^2 + 4)$
92. $(x + y)(x - y)(x^2 + y^2)$

In Exercises 93–96, find the product. (The expressions are not polynomials, but the formulas can still be used.)

93. $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$
94. $(5 + \sqrt{x})(5 - \sqrt{x})$
95. $(x - \sqrt{5})^2$
96. $(x + \sqrt{3})^2$

In Exercises 97–104, factor out the common factor.

97. $4x + 16$
98. $5y - 30$
99. $2x^3 - 6x$
100. $3z^3 - 6z^2 + 9z$
101. $3x(x - 5) + 8(x - 5)$
102. $3x(x + 2) - 4(x + 2)$
103. $(x + 3)^2 - 4(x + 3)$
104. $(5x - 4)^2 + (5x - 4)$

In Exercises 105–110, find the greatest common factor such that the remaining factors have only integer coefficients.

105. $\frac{1}{2}x + 4$
106. $\frac{1}{3}y + 5$
107. $\frac{1}{2}x^3 + 2x^2 - 5x$
108. $\frac{1}{3}y^4 - 5y^2 + 2y$
109. $\frac{2}{3}x(x - 3) - 4(x - 3)$
110. $\frac{4}{5}y(y + 1) - 2(y + 1)$

In Exercises 111–120, completely factor the difference of two squares.

111. $x^2 - 81$
112. $x^2 - 64$
113. $48y^2 - 27$
114. $50 - 98z^2$
115. $16x^2 - \frac{1}{9}$
116. $\frac{4}{25}y^2 - 64$
117. $(x - 1)^2 - 4$
118. $25 - (z + 5)^2$
119. $9u^2 - 4v^2$
120. $25x^2 - 16y^2$

In Exercises 121–132, factor the perfect square trinomial.

121. $x^2 - 4x + 4$
122. $x^2 + 10x + 25$
123. $4t^2 + 4t + 1$
124. $9x^2 - 12x + 4$
125. $25y^2 - 10y + 1$
126. $36y^2 - 108y + 81$
127. $9u^2 + 24uv + 16v^2$
128. $4x^2 - 4xy + y^2$
129. $x^2 - \frac{4}{3}x + \frac{4}{9}$
130. $z^2 + z + \frac{1}{4}$
131. $4x^2 - \frac{4}{3}x + \frac{1}{9}$
132. $9y^2 - \frac{3}{2}y + \frac{1}{16}$

In Exercises 133–144, factor the sum or difference of cubes.

133. $x^3 - 8$
134. $x^3 - 27$
135. $y^3 + 64$
136. $z^3 + 216$
137. $x^3 - \frac{8}{27}$
138. $y^3 + \frac{8}{125}$
139. $8t^3 - 1$
140. $27x^3 + 8$
141. $u^3 + 27v^3$
142. $64x^3 - y^3$
143. $(x + 2)^3 - y^3$
144. $(x - 3y)^3 - 8z^3$

In Exercises 145–158, factor the trinomial.

145. $x^2 + x - 2$
146. $x^2 + 5x + 6$
147. $s^2 - 5s + 6$
148. $t^2 - t - 6$
149. $20 - y - y^2$
150. $24 + 5z - z^2$
151. $x^2 - 30x + 200$
152. $x^2 - 13x + 42$
153. $3x^2 - 5x + 2$
154. $2x^2 - x - 1$
155. $5x^2 + 26x + 5$
156. $12x^2 + 7x + 1$
157. $-9z^2 + 3z + 2$
158. $-5u^2 - 13u + 6$

In Exercises 159–166, factor by grouping.

159. $x^3 - x^2 + 2x - 2$
160. $x^3 + 5x^2 - 5x - 25$
161. $2x^3 - x^2 - 6x + 3$
162. $5x^3 - 10x^2 + 3x - 6$
163. $6 + 2x - 3x^3 - x^4$
164. $x^5 + 2x^3 + x^2 + 2$
165. $6x^3 - 2x + 3x^2 - 1$
166. $8x^5 - 6x^2 + 12x^3 - 9$

In Exercises 167–172, factor the trinomial by grouping.

167. $3x^2 + 10x + 8$
168. $2x^2 + 9x + 9$
169. $6x^2 + x - 2$
170. $6x^2 - x - 15$
171. $15x^2 - 11x + 2$
172. $12x^2 - 13x + 1$

In Exercises 173–206, completely factor the expression.

173. $6x^2 - 54$
174. $12x^2 - 48$
175. $x^3 - x^2$
176. $x^3 - 4x^2$
177. $x^3 - 16x$
178. $x^3 - 9x$
179. $x^2 - 2x + 1$
180. $16 + 6x - x^2$
181. $1 - 4x + 4x^2$
182. $-9x^2 + 6x - 1$
183. $2x^2 + 4x - 2x^3$
184. $13x + 6 + 5x^2$
185. $\frac{1}{81}x^2 + \frac{2}{9}x - 8$
186. $\frac{1}{8}x^2 - \frac{1}{96}x - \frac{1}{16}$

187. $3x^3 + x^2 + 15x + 5$ **188.** $5 - x + 5x^2 - x^3$

189. $x^4 - 4x^3 + x^2 - 4x$ **190.** $3x^3 + x^2 - 27x - 9$

191. $\frac{1}{4}x^3 + 3x^2 + \frac{3}{4}x + 9$ **192.** $\frac{1}{5}x^3 + x^2 - x - 5$

193. $(t - 1)^2 - 49$ **194.** $(x^2 + 1)^2 - 4x^2$

195. $(x^2 + 8)^2 - 36x^2$ **196.** $2t^3 - 16$

197. $5x^3 + 40$ **198.** $4x(2x - 1) + (2x - 1)^2$

199. $5(3 - 4x)^2 - 8(3 - 4x)(5x - 1)$

200. $2(x + 1)(x - 3)^2 - 3(x + 1)^2(x - 3)$

201. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x)^3$

202. $7x(2)(x^2 + 1)(2x) - (x^2 + 1)^2(7)$

203. $3(x - 2)^2(x + 1)^4 + (x - 2)^3(4)(x + 1)^3$

204. $2x(x - 5)^4 - x^2(4)(x - 5)^3$

205. $5(x^6 + 1)^4(6x^5)(3x + 2)^3 + 3(3x + 2)^2(3)(x^6 + 1)^5$

206. $\frac{x^2}{2}(x^2 + 1)^4 - (x^2 + 1)^5$

In Exercises 207–212, completely factor the expression.

207. $x^4(4)(2x + 1)^3(2x) + (2x + 1)^4(4x^3)$

208. $x^3(3)(x^2 + 1)^2(2x) + (x^2 + 1)^3(3x^2)$

209. $(2x - 5)^4(3)(5x - 4)^2(5) + (5x - 4)^3(4)(2x - 5)^3(2)$

210. $(x^2 - 5)^3(2)(4x + 3)(4) + (4x + 3)^2(3)(x^2 - 5)^2(x^2)$

211. $\dfrac{(5x - 1)(3) - (3x + 1)(5)}{(5x - 1)^2}$

212. $\dfrac{(2x + 3)(4) - (4x - 1)(2)}{(2x + 3)^2}$

In Exercises 213–216, find all values of b for which the trinomial can be factored.

213. $x^2 + bx - 15$ **214.** $x^2 + bx - 12$

215. $x^2 + bx + 50$ **216.** $x^2 + bx + 24$

In Exercises 217–220, find two integer values of c such that the trinomial can be factored. (There are many correct answers.)

217. $2x^2 + 5x + c$ **218.** $3x^2 - 10x + c$

219. $3x^2 - x + c$ **220.** $2x^2 + 9x + c$

221. COST, REVENUE, AND PROFIT An electronics manufacturer can produce and sell x MP3 players per week. The total cost C (in dollars) of producing x MP3 players is $C = 73x + 25{,}000$, and the total revenue R (in dollars) is $R = 95x$.

(a) Find the profit P in terms of x.

(b) Find the profit obtained by selling 5000 MP3 players per week.

222. COMPOUND INTEREST After 3 years, an investment of \$1200 compounded annually at an interest rate r will yield an amount of $1200(1 + r)^3$.

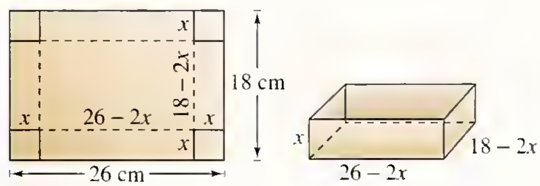
(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r shown in the table.

r	2%	3%	$3\frac{1}{2}$%	4%	$4\frac{1}{2}$%
$1200(1 + r)^3$					

(c) What conclusion can you make from the table?

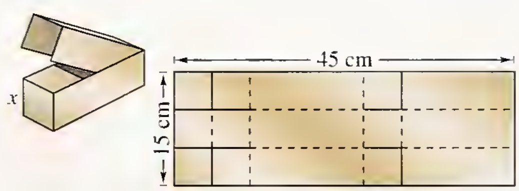
223. VOLUME OF A BOX A take-out fast-food restaurant is constructing an open box by cutting squares from the corners of a piece of cardboard that is 18 centimeters by 26 centimeters (see figure). The edge of each cut-out square is x centimeters.



(a) Find the volume of the box in terms of x.

(b) Find the volume when $x = 1$, $x = 2$, and $x = 3$.

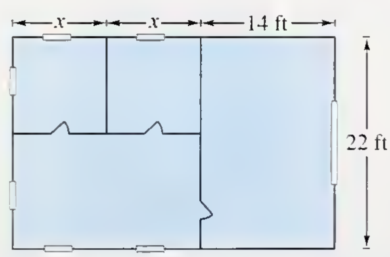
224. VOLUME OF A BOX An overnight shipping company is designing a closed box by cutting along the solid lines and folding along the broken lines on the rectangular piece of corrugated cardboard shown in the figure. The length and width of the rectangle are 45 centimeters and 15 centimeters, respectively.



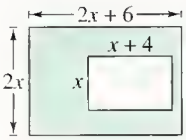
(a) Find the volume of the shipping box in terms of x.

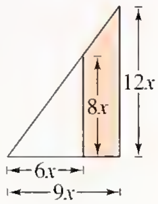
(b) Find the volume when $x = 3$, $x = 5$, and $x = 7$.

225. GEOMETRY Find a polynomial that represents the total number of square feet for the floor plan shown in the figure.



226. GEOMETRY Find the area of the shaded region in each figure. Write your result as a polynomial in standard form.

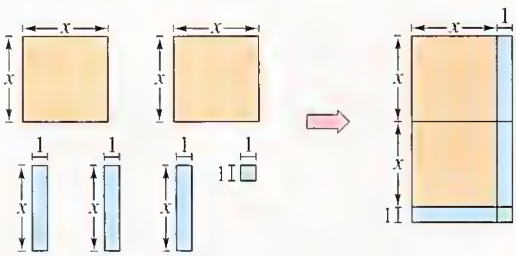
(a)


(b)


GEOMETRIC MODELING In Exercises 227–230, draw a "geometric factoring model" to represent the factorization. For instance, a factoring model for

$2x^2 + 3x + 1 = (2x + 1)(x + 1)$

is shown in the figure.



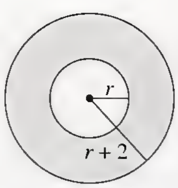
227. $3x^2 + 7x + 2 = (3x + 1)(x + 2)$

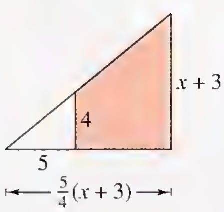
228. $x^2 + 4x + 3 = (x + 3)(x + 1)$

229. $2x^2 + 7x + 3 = (2x + 1)(x + 3)$

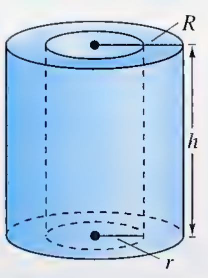
230. $x^2 + 3x + 2 = (x + 2)(x + 1)$

GEOMETRY In Exercises 231 and 232, write an expression in factored form for the area of the shaded portion of the figure.

231.


232.


233. GEOMETRY The cylindrical shell shown in the figure has a volume of $V = \pi R^2 h - \pi r^2 h$.



(a) Factor the expression for the volume.

(b) From the result of part (a), show that the volume is 2π (average radius)(thickness of the shell)h.

234. CHEMISTRY The rate of change of an autocatalytic chemical reaction is $kQx - kx^2$, where Q is the amount of the original substance, x is the amount of substance formed, and k is a constant of proportionality. Factor the expression.

EXPLORATION

TRUE OR FALSE? In Exercises 235–238, determine whether the statement is true or false. Justify your answer.

235. The product of two binomials is always a second-degree polynomial.

236. The sum of two binomials is always a binomial.

237. The difference of two perfect squares can be factored as the product of conjugate pairs.

238. The sum of two perfect squares can be factored as the binomial sum squared.

239. Find the degree of the product of two polynomials of degrees m and n.

240. Find the degree of the sum of two polynomials of degrees m and n if $m < n$.

241. THINK ABOUT IT When the polynomial $-x^3 + 3x^2 + 2x - 1$ is subtracted from an unknown polynomial, the difference is $5x^2 + 8$. If it is possible, find the unknown polynomial.

242. LOGICAL REASONING Verify that $(x + y)^2$ is not equal to $x^2 + y^2$ by letting $x = 3$ and $y = 4$ and evaluating both expressions. Are there any values of x and y for which $(x + y)^2 = x^2 + y^2$? Explain.

243. Factor $x^{2n} - y^{2n}$ as completely as possible.

244. Factor $x^{3n} + y^{3n}$ as completely as possible.

245. Give an example of a polynomial that is prime with respect to the integers.

246. CAPSTONE A third-degree polynomial and a fourth-degree polynomial are added.

(a) Can the sum be a fourth-degree polynomial? Explain or give an example.

(b) Can the sum be a second-degree polynomial? Explain or give an example.

(c) Can the sum be a seventh-degree polynomial? Explain or give an example.

(d) After adding the two polynomials and factoring the sum, you obtain a polynomial that is in factored form. Explain what is meant by saying that a polynomial is in factored form.

A.4 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. The set of real numbers for which an algebraic expression is defined is the ________ of the expression.
2. The quotient of two algebraic expressions is a fractional expression and the quotient of two polynomials is a ________ ________.
3. Fractional expressions with separate fractions in the numerator, denominator, or both are called ________ fractions.
4. To simplify an expression with negative exponents, it is possible to begin by factoring out the common factor with the ________ exponent.
5. Two algebraic expressions that have the same domain and yield the same values for all numbers in their domains are called ________.
6. An important rational expression, such as $\frac{(x+h)^2 - x^2}{h}$, that occurs in calculus is called a ________ ________.

SKILLS AND APPLICATIONS

In Exercises 7–22, find the domain of the expression.

7. $3x^2 - 4x + 7$

8. $2x^2 + 5x - 2$

9. $4x^3 + 3, \quad x \geq 0$

10. $6x^2 - 9, \quad x > 0$

11. $\dfrac{1}{3 - x}$

12. $\dfrac{x + 6}{3x + 2}$

13. $\dfrac{x^2 - 1}{x^2 - 2x + 1}$

14. $\dfrac{x^2 - 5x + 6}{x^2 - 4}$

15. $\dfrac{x^2 - 2x - 3}{x^2 - 6x + 9}$

16. $\dfrac{x^2 - x - 12}{x^2 - 8x + 16}$

17. $\sqrt{x + 7}$

18. $\sqrt{4 - x}$

19. $\sqrt{2x - 5}$

20. $\sqrt{4x + 5}$

21. $\dfrac{1}{\sqrt{x - 3}}$

22. $\dfrac{1}{\sqrt{x + 2}}$

In Exercises 23 and 24, find the missing factor in the numerator such that the two fractions are equivalent.

23. $\dfrac{5}{2x} = \dfrac{5(\quad)}{6x^2}$

24. $\dfrac{3}{4} = \dfrac{3(\quad)}{4(x + 1)}$

In Exercises 25–42, write the rational expression in simplest form.

25. $\dfrac{15x^2}{10x}$

26. $\dfrac{18y^2}{60y^5}$

27. $\dfrac{3xy}{xy + x}$

28. $\dfrac{2x^2y}{xy - y}$

29. $\dfrac{4y - 8y^2}{10y - 5}$

30. $\dfrac{9x^2 + 9x}{2x + 2}$

31. $\dfrac{x - 5}{10 - 2x}$

32. $\dfrac{12 - 4x}{x - 3}$

33. $\dfrac{y^2 - 16}{y + 4}$

34. $\dfrac{x^2 - 25}{5 - x}$

35. $\dfrac{x^3 + 5x^2 + 6x}{x^2 - 4}$

36. $\dfrac{x^2 + 8x - 20}{x^2 + 11x + 10}$

37. $\dfrac{y^2 - 7y + 12}{y^2 + 3y - 18}$

38. $\dfrac{x^2 - 7x + 6}{x^2 + 11x + 10}$

39. $\dfrac{2 - x + 2x^2 - x^3}{x^2 - 4}$

40. $\dfrac{x^2 - 9}{x^3 + x^2 - 9x - 9}$

41. $\dfrac{z^3 - 8}{z^2 + 2z + 4}$

42. $\dfrac{y^3 - 2y^2 - 3y}{y^3 + 1}$

43. *Error Analysis* Describe the error.

$$\frac{5x^3}{2x^3 + 4} = \frac{5\cancel{x^3}}{2\cancel{x^3} + 4} = \frac{5}{2 + 4} = \frac{5}{6}$$

44. *Error Analysis* Describe the error.

$$\frac{x^3 + 25x}{x^2 - 2x - 15} = \frac{x(x^2 + 25)}{(x - 5)(x + 3)} = \frac{x(x + 5)(x - 5)}{(x - 5)(x + 3)} = \frac{x(x + 5)}{x + 3}$$

In Exercises 45 and 46, complete the table. What can you conclude?

45.

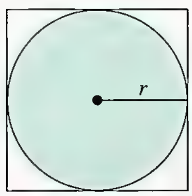
x	0	1	2	3	4	5	6
$\dfrac{x^2 - 2x - 3}{x - 3}$							
$x + 1$							

46.

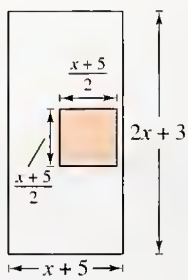
x	0	1	2	3	4	5	6
$\dfrac{x - 3}{x^2 - x - 6}$							
$\dfrac{1}{x + 2}$							

GEOMETRY In Exercises 47 and 48, find the ratio of the area of the shaded portion of the figure to the total area of the figure.

47.



48.



In Exercises 49–56, perform the multiplication or division and simplify.

49. $\dfrac{5}{x - 1} \cdot \dfrac{x - 1}{25(x - 2)}$

50. $\dfrac{x + 13}{x^3(3 - x)} \cdot \dfrac{x(x - 3)}{5}$

51. $\dfrac{r}{r - 1} \div \dfrac{r^2}{r^2 - 1}$

52. $\dfrac{4y - 16}{5y + 15} \div \dfrac{4 - y}{2y + 6}$

53. $\dfrac{t^2 - t - 6}{t^2 + 6t + 9} \cdot \dfrac{t + 3}{t^2 - 4}$

54. $\dfrac{x^2 + xy - 2y^2}{x^3 + x^2y} \cdot \dfrac{x}{x^2 + 3xy + 2y^2}$

55. $\dfrac{x^2 - 36}{x} \div \dfrac{x^3 - 6x^2}{x^2 + x}$

56. $\dfrac{x^2 - 14x + 49}{x^2 - 49} \div \dfrac{3x - 21}{x + 7}$

In Exercises 57–68, perform the addition or subtraction and simplify.

57. $6 - \dfrac{5}{x+3}$

58. $\dfrac{3}{x-1} - 5$

59. $\dfrac{5}{x-1} + \dfrac{x}{x-1}$

60. $\dfrac{2x-1}{x+3} + \dfrac{1-x}{x+3}$

61. $\dfrac{3}{x-2} + \dfrac{5}{2-x}$

62. $\dfrac{2x}{x-5} - \dfrac{5}{5-x}$

63. $\dfrac{4}{2x+1} - \dfrac{x}{x+2}$

64. $\dfrac{2}{x-3} + \dfrac{5x}{3x+4}$

65. $\dfrac{1}{x^2-x-2} - \dfrac{x}{x^2-5x+6}$

66. $\dfrac{2}{x^2-x-2} + \dfrac{10}{x^2+2x-8}$

67. $-\dfrac{1}{x} + \dfrac{2}{x^2+1} + \dfrac{1}{x^3+x}$

68. $\dfrac{2}{x+1} + \dfrac{2}{x-1} + \dfrac{1}{x^2-1}$

ERROR ANALYSIS In Exercises 69 and 70, describe the error.

69. $\dfrac{x+4}{x+2} - \dfrac{3x-8}{x+2} = \dfrac{x+4-3x-8}{x+2}$

$= \dfrac{-2x-4}{x+2} = \dfrac{-2(x+2)}{x+2} = -2$

70. $\dfrac{6-x}{x(x+2)} + \dfrac{x+2}{x^2} + \dfrac{8}{x^2(x+2)}$

$= \dfrac{x(6-x)+(x+2)^2+8}{x^2(x+2)}$

$= \dfrac{6x-x^2+x^2+4+8}{x^2(x+2)}$

$= \dfrac{6(x+2)}{x^2(x+2)} = \dfrac{6}{x^2}$

In Exercises 71–76, simplify the complex fraction.

71. $\dfrac{\left(\dfrac{x}{2}-1\right)}{(x-2)}$

72. $\dfrac{(x-4)}{\left(\dfrac{x}{4}-\dfrac{4}{x}\right)}$

73. $\dfrac{\left[\dfrac{x^2}{(x+1)^2}\right]}{\left[\dfrac{x}{(x+1)^3}\right]}$

74. $\dfrac{\left(\dfrac{x^2-1}{x}\right)}{\left[\dfrac{(x-1)^2}{x}\right]}$

75. $\dfrac{\left(\sqrt{x}-\dfrac{1}{2\sqrt{x}}\right)}{\sqrt{x}}$

76. $\dfrac{\left(\dfrac{t^2}{\sqrt{t^2+1}}-\sqrt{t^2+1}\right)}{t^2}$

In Exercises 77–82, factor the expression by removing the common factor with the smaller exponent.

77. $x^5 - 2x^{-2}$

78. $x^5 - 5x^{-3}$

79. $x^2(x^2+1)^{-5} - (x^2+1)^{-4}$

80. $2x(x-5)^{-3} - 4x^2(x-5)^{-4}$

81. $2x^2(x-1)^{1/2} - 5(x-1)^{-1/2}$

82. $4x^3(2x-1)^{3/2} - 2x(2x-1)^{-1/2}$

In Exercises 83 and 84, simplify the expression.

83. $\dfrac{3x^{1/3}-x^{-2/3}}{3x^{-2/3}}$

84. $\dfrac{-x^3(1-x^2)^{-1/2}-2x(1-x^2)^{1/2}}{x^4}$

In Exercises 85–88, simplify the difference quotient.

85. $\dfrac{\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)}{h}$

86. $\dfrac{\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]}{h}$

87. $\dfrac{\left(\dfrac{1}{x+h-4}-\dfrac{1}{x-4}\right)}{h}$

88. $\dfrac{\left(\dfrac{x+h}{x+h+1}-\dfrac{x}{x+1}\right)}{h}$

In Exercises 89–94, simplify the difference quotient by rationalizing the numerator.

89. $\dfrac{\sqrt{x+2}-\sqrt{x}}{2}$

90. $\dfrac{\sqrt{z-3}-\sqrt{z}}{3}$

91. $\dfrac{\sqrt{t+3}-\sqrt{3}}{t}$

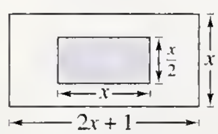
92. $\dfrac{\sqrt{x+5}-\sqrt{5}}{x}$

93. $\dfrac{\sqrt{x+h+1}-\sqrt{x+1}}{h}$

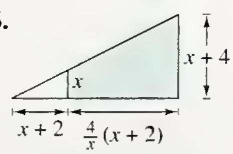
94. $\dfrac{\sqrt{x+h-2}-\sqrt{x-2}}{h}$

PROBABILITY In Exercises 95 and 96, consider an experiment in which a marble is tossed into a box whose base is shown in the figure. The probability that the marble will come to rest in the shaded portion of the box is equal to the ratio of the shaded area to the total area of the figure. Find the probability.

95.



96.



97. RATE A digital copier copies in color at a rate of 50 pages per minute.

(a) Find the time required to copy one page.

(b) Find the time required to copy x pages.

(c) Find the time required to copy 120 pages.

98. RATE After working together for t hours on a common task, two workers have done fractional parts of the job equal to $t/3$ and $t/5$, respectively. What fractional part of the task has been completed?

FINANCE In Exercises 99 and 100, the formula that approximates the annual interest rate r of a monthly installment loan is given by

$$r = \frac{\left[\dfrac{24(NM - P)}{N}\right]}{\left(P + \dfrac{NM}{12}\right)}$$

where N is the total number of payments, M is the monthly payment, and P is the amount financed.

99. (a) Approximate the annual interest rate for a four-year car loan of \$20,000 that has monthly payments of \$475.

(b) Simplify the expression for the annual interest rate r, and then rework part (a).

100. (a) Approximate the annual interest rate for a five-year car loan of \$28,000 that has monthly payments of \$525.

(b) Simplify the expression for the annual interest rate r, and then rework part (a).

101. REFRIGERATION When food (at room temperature) is placed in a refrigerator, the time required for the food to cool depends on the amount of food, the air circulation in the refrigerator, the original temperature of the food, and the temperature of the refrigerator. The model that gives the temperature of food that has an original temperature of 75°F and is placed in a 40°F refrigerator is

$$T = 10\left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$$

where T is the temperature (in degrees Fahrenheit) and t is the time (in hours).

(a) Complete the table.

t	0	2	4	6	8	10	12
T							

t	14	16	18	20	22
T					

(b) What value of T does the mathematical model appear to be approaching?

102. INTERACTIVE MONEY MANAGEMENT The table shows the projected numbers of U.S. households (in millions) banking online and paying bills online from 2002 through 2007. (Source: eMarketer; Forrester Research)

Year	Banking	Paying Bills
2002	21.9	13.7
2003	26.8	17.4
2004	31.5	20.9
2005	35.0	23.9
2006	40.0	26.7
2007	45.0	29.1

Mathematical models for these data are

$$\text{Number banking online} = \frac{-0.728t^2 + 23.81t - 0.3}{-0.049t^2 + 0.61t + 1.0}$$

and

$$\text{Number paying bills online} = \frac{4.39t + 5.5}{0.002t^2 + 0.01t + 1.0}$$

where t represents the year, with $t = 2$ corresponding to 2002.

(a) Using the models, create a table to estimate the projected numbers of households banking online and the projected numbers of households paying bills online for the given years.

(b) Compare the values given by the models with the actual data.

(c) Determine a model for the ratio of the projected number of households paying bills online to the projected number of households banking online.

(d) Use the model from part (c) to find the ratios for the given years. Interpret your results.

EXPLORATION

TRUE OR FALSE? In Exercises 103 and 104, determine whether the statement is true or false. Justify your answer.

103. $\dfrac{x^{2n} - 1^{2n}}{x^n - 1^n} = x^n + 1^n$

104. $\dfrac{x^2 - 3x + 2}{x - 1} = x - 2$, for all values of x

105. THINK ABOUT IT How do you determine whether a rational expression is in simplest form?

106. CAPSTONE In your own words, explain how to divide rational expressions.